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Rolling tachyons and decaying branes

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Abstract: We present new rolling tachyon solutions describing the classical decay of D-branes. Our methods are simpler than those appearing in recent works, yet our results are exact in classical string theory. The role of pressure in the decay is studied using tachyon profiles with spatial variation. In this case the final state involves an array of codimension one D-branes rather than static, pressureless tachyon matter.

Keywords: D-branes, Tachyon Condensation.

Contents

1. Introduction and summary

An appealing way to generate time dependent configurations in string theory is to consider the classical decay of unstable systems of D-branes, pictured as a tachyon field rolling down a potential, towards a stable minimum. This system is promising from the viewpoint of studying time-dependence in string theory since the non-trivial dynamics is confined to the open string sector. Moreover, this setting provides a natural arena for discussing important cosmological ideas, such as inflation, and the beginning of time.

The quantitative study of rolling tachyons was initiated recently by A. Sen [1]–[5]. It involves deforming the world sheet conformal field theory (CFT) of the unstable D-brane by an exactly marginal, time dependent tachyon profile. The deformed CFT is an exact classical background in string theory, interpreted as the classical decay of the unstable D-brane. Although several key results have been obtained, this approach is still in its infancy and central questions remain:

- In the classical approximation, the string coupling constant is strictly vanishing. However, it is not clear that the implied limit is smooth. The system with a small, but non-zero string coupling may differ *qualitatively* from the system with vanishing coupling. In such a scenario, the classical approximation is misleading. This concern is fueled by the somewhat mysterious role of closed strings in tachyon condensation: after the decay of the unstable D-brane only closed strings remain in the spectrum; yet the brane cannot decay into closed strings if the coupling is strictly vanishing.

- A tachyon profile with spatial momentum $\vec{k}$ has effective mass[1] $m_t^2 = -1 + \vec{k}^2$ and so is unstable for any $|\vec{k}| < 1$, indicating that *all* wave lengths play some role in the decay. Indeed, in quantum field theory it is well understood that tachyon condensation is a process where the longest wave lengths dominate, but all wave lengths participate, and the decay is definitely inhomogeneous. These results are best known in the context of cosmological inflation [7, 8], but they are valid also for tachyon condensation in string theory [9]. They indicate that spatially inhomogenous modes are important also for rolling tachyons.[2]

In this paper we study several new tachyon profiles with the goal of shedding light on these questions. The deformations we consider are actually technically simpler than those previously studied. We are therefore able to avoid the full machinery of boundary states and instead carry out the computations using elementary methods. The approach taken here complements the one taken in previous papers on rolling tachyons and may offer some conceptual advantages. As discussed in section 5, the new spatially homogenous profile, given below as (1.1), has a topology that is different from previous examples. In addition to this decay, in which the rolling tachyon has no spatial dependence, we study an exactly soluble example with spatially varying tachyon profile, where we can follow the decay to its inhomogeneous final state.

The simplest of the new profiles studied in this paper is

$$T(X) = \lambda e^{X^0} . \tag{1.1}$$

This can be interpreted as simultaneously displacing and giving a velocity to the tachyon, i.e. imposing the initial condition $T(X^0 = 0) = \partial_t T(X^0 = 0) = \lambda$. An alternative, and better, spacetime interpretation is that of a perturbation at $X^0 = -\infty$.[3] Since this disturbance is automatically infinitesimal, the profile (1.1) seems to be a particularly clean example of a rolling tachyon.

The profile (1.1) is also particularly simple from a technical point of view. The simplest exactly marginal deformations of a world sheet CFT are generated by the vertex operators

$$V(X) = e^{ikX} : \qquad k^2 = k_0^2 - \vec{k}^2 = -1 . \tag{1.2}$$

Of course these operator cannot usually be added to the world sheet action because they correspond to complex potentials. The standard remedy is to add also the conjugate

[1] We use units such that $\alpha' = 1$.

[2] Other discussions of spatial variation include [4] and [10]–[12].

[3] This was the point of view taken in the recent talk by Strominger [13].

operator and so consider perturbations of the form

$$T(\vec{X}) = \lambda \cos(\vec{k} \cdot \vec{X}) = \frac{\lambda}{2}(e^{i\vec{k}\cdot\vec{X}} + e^{-i\vec{k}\cdot\vec{X}})\,; \qquad \vec{k}^2 = 1\,. \tag{1.3}$$

After analytical continuation this leads to Sen's profile $T(X) = \lambda \cosh(X^0)$. An alternative procedure, exploited in this paper, is to note that in the special case $k_0 = -i$, $\vec{k} = 0$ the vertex operator (1.2) is in fact real, and so we can consider (1.1) directly, without adding the complex conjugate. This is much simpler because, in the case of (1.3), complications arise from the cross-terms between the two exponentials.

As explained above, it is important to study rolling tachyons with spatially varying profiles. Such profiles are generally quite complicated to analyze in the full CFT; but in the case of the profile

$$T(\vec{X}) = \lambda e^{X^0/\sqrt{2}} \cos(\vec{k} \cdot \vec{X})\,; \qquad \vec{k}^2 = \frac{1}{2}\,, \tag{1.4}$$

the study simplifies dramatically (similar, but more complicated tachyon profiles were discussed in [4]). Indeed, this profile is a linear combination of two vertex operators of the form (1.2). Crucially, these vertex operators commute, in contrast to those appearing in (1.3). Thus the theory with the profile (1.4) essentially reduces to two copies of (1.1).

Having solved the theory with the tachyon profile (1.4) we find the coupling to the energy momentum tensor for all times X^0. The energy momentum tensor exhibits qualitatively different behavior from the spatially homogenous case. It develops codimension one singularities in finite time. These singularities can be interpreted as an array of (excited) D-branes. This result is consistent with the expectation that final states will be spatially inhomogeneous for generic decay channels. Of course, the profile that we can actually solve (1.4) is actually quite special. Presumably that is why the final state in this example, although spatially inhomogeneous, is as finely tuned as a perfect array of unstable branes. In a realistic, semi-classical, analysis one would choose as initial state for the brane some wave-packet localized near the top of the tachyon potential and the full decay process would be described as an average, in a precise sense, of all the initial conditions represented by this wave packet. One would expect this final state to be dominated by generic, spatially dependent, configurations which, to the extent they can be described as a perfect fluid, certainly would have pressure.

This paper is organized as follows. In section 2 we explain our approach to rolling tachyons and carry out the details for the profile (1.1). In section 3 we consider the spatially inhomogeneous case (1.4) and discuss the lessons for the full decay process, when all spatial variations are included. In section 4, we extend these results to the superstring case. Finally, in section 5, we discuss the topology of our profiles, their relations to previous works, and construct the boundary states corresponding to our solutions.

2. Rolling tachyons

The strategy for treating rolling tachyons is to deform the world sheet CFT of an unstable D-brane by an exactly marginal operator and interpret the deformed CFT as a time dependent solution to the classical string equations of motion. Instead of studying the

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system in terms of a boundary state of the closed string theory (which was Sen's approach in [1, 2, 4]), we will primarily work with open strings, equating the disk partition function with space-time action, following the analysis of static tachyon configuations in boundary string field theory [14].

2.1 Generalities

According to the σ-model approach to string theory, the space-time action is given by the partition function of the world-sheet theory, with the world-sheet couplings interpreted as spacetime fields [15]. Thus, in the open string sector,

$$S[\lambda_i] \propto Z_{\text{disk}}(\lambda_i) = \int [dX^\mu] e^{-I_{\text{bulk}} - I_{\text{bndy}}} , \tag{2.1}$$

where Z_{disk} is the disk partition function, λ_i are exactly marginal couplings for the boundary operators, and

$$I_{\text{bulk}} = \frac{1}{2\pi} \int_D d^2z \; \eta_{\mu\nu} \; \partial X^\mu \bar{\partial} X^\nu , \tag{2.2}$$

$$I_{\text{bndy}} = \int_{\partial D} dt \; T(X) + \cdots , \tag{2.3}$$

where D is the unit disk and ∂D is its boundary. T is the tachyon field and the $\cdots$ indicate other marginal boundary perturbations. This procedure is similar in spirit to the boundary string field theory approach to tachyon condensation [14], although here we limit ourselves to marginal perturbations.

Our formulae above are written for euclidean spacetimes, as well as euclidean world-sheets. The time-dependence is then taken into account by including the minkowskian metric $\eta^{\mu\nu} = (-, +, \cdots, +)$ when contracting the temporal fields X^0. This procedure is motivated by analytical continuation, as in Sen's computations using boundary states and cubic string field theory. This type of analytical continuation is clearly not completely satisfying; indeed, the precise relation between lorentzian and euclidean signature is one of the main unsettled questions facing most approaches to time dependence in string theory. On the other hand, the analytical continuation gives physically reasonable results in many examples, including those considered here; so it presumably captures important aspects of the problem.

It will be useful to write the action S as a space-time integral over a lagrangian density. To this end, we split X^μ into a constant and a varying part, $X^\mu = x^\mu + X'^\mu$ and write

$$\begin{aligned} S &\propto \int [dX^\mu] e^{-I_{\text{bulk}} - I_{\text{bndy}}} \\ &= \int d^p x \sqrt{-g} \int [dX'^\mu] e^{-\frac{1}{2\pi}\int_D d^2z \; g_{\mu\nu} \; \partial X'^\mu \bar{\partial} X'^\nu - I_{\text{bndy}}} , \end{aligned} \tag{2.4}$$

where, in the second line, we have made the obvious generalization of the expression to curved space. In this paper we are primarily considering the coupling to gravity, to explore the time-evolution of an unstable D-brane in a flat background.[4] From the spacetime action

[4]It would be interesting to apply our methods for couplings to more massive strings as well, in an effort to illuminate the problems discussed in [16]. See also [17] for the coupling to the closed string.

we form the energy-momentum tensor $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$ and use $\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}$ to find

$$T_{\mu\nu}(x) = K\left(\mathcal{B}(x)\ \eta_{\mu\nu} + \mathcal{A}_{\mu\nu}(x)\right)\ , \tag{2.5}$$

in flat space. Here K is an overall normalization constant and

$$\mathcal{B}(x) = \int [dX'^{\mu}] e^{-\frac{1}{2\pi}\int_D d^2z\ \eta_{\mu\nu}\ \partial X'^{\mu}\bar{\partial}X'^{\nu} - I_{\rm bndy}}\ , \tag{2.6}$$

$$\begin{aligned}\mathcal{A}^{\mu\nu}(x) &= 2\int [dX'^{\mu}]\int \frac{d^2z}{2\pi}\ \partial X^{\mu}\bar{\partial}X^{\nu} e^{-\frac{1}{2\pi}\int_D d^2z\ \eta_{\mu\nu}\ \frac{1}{2\pi}\partial X'^{\mu}\bar{\partial}X'^{\nu} - I_{\rm bndy}} \\ &= 2\int [dX'^{\mu}]\ \partial X^{\mu}(0)\bar{\partial}X^{\nu}(0) e^{-\frac{1}{2\pi}\int_D d^2z\ \eta_{\mu\nu}\ \partial X'^{\mu}\bar{\partial}X'^{\nu} - I_{\rm bndy}}\ .\end{aligned} \tag{2.7}$$

In the second line of (2.7) we fixed the position of the vertex operator and used $\int \frac{d^2z}{2\pi} = \frac{1}{\pi}A(D) = 1$ for the unit disc. The expression (2.5) for the energy-momentum tensor was previously derived by Sen [1] using BRST invariance of the corresponding boundary state. In the following we consider various $T(X)$, corresponding to tachyon profiles with specific space-time dependence. To determine the energy-momentum tensor for each profile we need to compute $\mathcal{A}^{\mu\nu}$ and $\mathcal{B}$.

2.2 Spatially homogenous decay

In this section we study the tachyon profile

$$T(X) = \lambda e^{X^0}\ . \tag{2.8}$$

This is an exactly marginal deformation of the CFT and so an exact solution to the classical string equations of motion. It is interpreted in spacetime as a perturbation at $X^0 = -\infty$, displacing the tachyon infinitesimally from the unstable maximum of the potential. Alternatively, this profile corresponds to kicking the tachyon from $T(X^0 = 0) = \lambda$, with velocity $\partial_t T(X^0 = 0) = \lambda$.

To determine the stress tensor $T_{\mu\nu}$ for this tachyonic profile, we need to compute the functions $\mathcal{B}(x^0)$ and $\mathcal{A}_{\mu\nu}(x^0)$.

2.2.1 $\mathcal{B}(x^0)$

The function $\mathcal{B}(x^0)$ is the disk partition function, except that zero modes remain unintegrated. Using $\langle\cdots\rangle$ as symbols for expectation values on the disc we have the perturbative expansion

$$\begin{aligned}\mathcal{B}(x^0) &= \langle e^{-I_{\rm bndy}(x+X')}\rangle = \langle e^{-\lambda e^{x^0}\int dt\ e^{X'^0}}\rangle\ , \end{aligned}\tag{2.9}$$

$$= \sum_{n=0}^{\infty}\frac{(-2\pi\lambda e^{x^0})^n}{n!}\int\frac{dt_1}{2\pi}\cdots\frac{dt_n}{2\pi}\langle e^{X'^0(t_1)}\cdots e^{X'^0(t_n)}\rangle\ , \tag{2.10}$$

The Green's function on the unit disk with Neumann boundary conditions is

$$G^{\mu\nu}(z,z') = \langle X^{\mu}(z)X^{\nu}(z')\rangle = \eta^{\mu\nu}(-\log|z-z'| - \log|z\bar{z}'-1|)\ , \tag{2.11}$$

so, taking $z_i = e^{it_i}$, we find

$$\langle e^{X'^0(t_1)} \cdots e^{X'^0(t_n)} \rangle = \prod_{i<j} |e^{it_i} - e^{it_j}|^2 = 4^{n(n-1)/2} \prod_{i<j} \sin^2 \left(\frac{t_i - t_j}{2} \right) . \tag{2.12}$$

The integrals in (2.10) give[5]

$$\int \frac{dt_1}{2\pi} \cdots \frac{dt_n}{2\pi} \, 4^{n(n-1)/2} \prod_{i<j} \sin^2 \left(\frac{t_i - t_j}{2} \right) = n! \,, \tag{2.15}$$

and the final result for $\mathcal{B}(x^0)$ becomes

$$\mathcal{B}(x^0) = \sum_{n=0}^{\infty} \frac{(-2\pi\lambda e^{x^0})^n}{n!} \, n! = f(x^0) \,, \tag{2.16}$$

where

$$f(x^0) \equiv \frac{1}{1 + 2\pi\lambda e^{x^0}} \,. \tag{2.17}$$

The summation of the perturbative series is clearly justified for couplings within the radius of convergence $|\lambda| < \frac{1}{2\pi} e^{-x^0}$. The regime of validity may be extended by analytical continuation to include all positive λ. The precise justification for this extension is an interesting question that deserves further study.

2.2.2 $\mathcal{A}^{\mu\nu}$

The $\mathcal{A}^{\mu\nu}$ are proportional to expectation values of graviton vertex operators $:\partial X^\mu \bar{\partial} X^\nu:$, where the normal ordering symbol $:\ \ :$ indicate that the divergent pieces have been subtracted as $z \to z'$

$$:\partial X^\mu(z) \bar{\partial} X^\nu(z'): \ = \partial X^\mu(z) \bar{\partial} X^\nu(z') + \eta^{\mu\nu} \partial \bar{\partial}' \log |z - z'| \,. \tag{2.18}$$

For the purpose of our calculation, it is useful to define another kind of normal ordering symbol ${}^\circ_\circ \ {}^\circ_\circ$ where we subtract the full Green's function (2.11), *viz*

$${}^\circ_\circ \partial X^\mu(z) \bar{\partial} X^\nu(z') {}^\circ_\circ \ = \partial X^\mu(z) \bar{\partial} X^\nu(z') - \partial \bar{\partial}' G^{\mu\nu}(z, z') \,, \tag{2.19}$$

including the contribution from the image charge. The two normal orderings are related as

$$:\partial X^\mu(0) \bar{\partial} X^\nu(0): \ = {}^\circ_\circ \partial X^\mu(0) \bar{\partial} X^\nu(0) {}^\circ_\circ + \frac{1}{2} \eta^{\mu\nu} \,. \tag{2.20}$$

---

[5] These integrals can be derived by considering an integration over $U(n)$ matrices and using the known result that the $U(n)$ Haar measure dU, when expressed in terms of the eigenvalues, becomes

$$\frac{1}{\mathrm{vol}\, U(n)} \int dU = \frac{1}{n!} \int \prod_i \frac{dt_i}{2\pi} \Delta^2(t) \,, \tag{2.13}$$

where $\Delta(t)$ is the relevant Vandermonde determinant for $U(n)$ matrices

$$\Delta(t) = \prod_{i<j} 2 \sin \left(\frac{t_i - t_j}{2} \right) . \tag{2.14}$$

By noticing that the LHS in (2.13) is 1, the integral in (2.15) follows immediately.

Now we are ready to calculate $\mathcal{A}^{\mu\nu}$. For $i,j \neq 0$ we have

$$
\begin{aligned}
\mathcal{A}^{ij}(x) &= 2\langle :\partial X^i(0)\bar{\partial} X^j(0): e^{-\lambda e^{x^0}\int dt\ e^{X'^0}}\rangle \\
&= 2\left\langle \left({}^\circ_\circ\partial X^i(0)\bar{\partial} X^j(0){}^\circ_\circ + \frac{\delta^{ij}}{2}\right) e^{-\lambda e^{x^0}\int dt\ e^{X'^0}}\right\rangle \\
&= \delta^{ij}\ f(x^0)\,.
\end{aligned} \tag{2.21}
$$

In going from the second to the third line, the normal ordered term between ${}^\circ_\circ\ {}^\circ_\circ$ gives no contribution, and the term proportional to δ^{ij} is exactly the same as $\mathcal{B}(x^0)$ computed earlier.

The calculation of $\mathcal{A}^{00}$ is a bit more involved.

$$
\begin{aligned}
\mathcal{A}^{00} &= 2\langle :\partial X^0(0)\bar{\partial} X^0(0): e^{-\lambda\int dt e^{X^0}}\rangle \\
&= 2\langle {}^\circ_\circ\partial X^0(0)\bar{\partial} X^0(0){}^\circ_\circ e^{-\lambda e^{x^0}\int dt e^{X'^0}}\rangle - f(x^0) \\
&= 2\sum_n \frac{(-2\pi\lambda e^{x^0})^n}{n!}\langle {}^\circ_\circ\partial X^0(0)\bar{\partial} X^0(0){}^\circ_\circ \prod_{i=1}^n \int \frac{dt_i}{2\pi} e^{X^0(e^{it_i})}\rangle - f(x^0)\,.
\end{aligned} \tag{2.22}
$$

The correlation function in (2.22) yields

$$
\langle {}^\circ_\circ\partial X^0(z)\bar{\partial} X^0(\bar{z}){}^\circ_\circ \prod_{i=1}^n : e^{X(w_i)}:\rangle = \prod_{i<j}|w_i - w_j|^2 \sum_{i,j}\left(\frac{1}{z-w_i}\right)\left(\frac{1}{\bar{z}-\bar{w}_j}\right)\delta_{n>0}\,, \tag{2.23}
$$

which, for $z=0$ and $w_j = e^{it_j}$ gives

$$
\begin{aligned}
\prod_{i=1}^n \int \frac{dt_i}{2\pi}\langle {}^\circ_\circ\partial X^0(0)\bar{\partial} X^0(0){}^\circ_\circ \prod_{i=1}^n : e^{X(e^{it_i})}:\rangle &= \prod_{i=1}^n \int \frac{dt_i}{2\pi}\prod_{i<j}|e^{it_i} - e^{it_j}|^2 \sum_{i,j} e^{-i(t_i-t_j)} \\
&= \prod_{i=1}^n \int \frac{dt_i}{2\pi}\prod_{i<j} 2\sin^2\left(\frac{t_i - t_j}{2}\right)\times \\
&\quad \times\left(n + 2\sum_{i<j}\cos(t_i - t_j)\right).
\end{aligned}
$$

The integral is

$$
\prod_{i=1}^n \int \frac{dt_i}{2\pi}\prod_{i<j} 2\sin^2\left(\frac{t_i - t_j}{2}\right)\left(n + 2\sum_{i<j}\cos(t_i - t_j)\right) = n!\,, \tag{2.24}
$$

and (2.22) becomes

$$
\mathcal{A}^{00} = f(x^0) - 2\,. \tag{2.25}
$$

Collecting the various results we find the stress tensor for the tachyon profile (2.8)

$$
T_{00} = K(-\mathcal{B}(x^0) + \mathcal{A}_{00}(x^0)) = -\mathcal{T}_p\,, \qquad T_{ij} = K(\mathcal{B}(x^0) + \mathcal{A}_{ij}(x^0)) = \delta_{ij}\mathcal{T}_p f(x^0)\,. \tag{2.26}
$$

We have determined the normalization constant $K = \frac{1}{2}\mathcal{T}_p$ by comparison with the static limit $\lambda = 0$. As expected, T_{00} is independent of x_0, which is just the statement of conservation of energy. Moreover, $T_{ij} \to 0$ as $x^0 \to \infty$, so the pressure vanishes in this limit, i.e. the decay product is pressureless tachyon matter, as in [2].

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3. Spatially inhomogenous decay

We will now investigate the spatially inhomogenous decay. A spatially inhomogenous profile $T(X) = 2\lambda e^{\omega X^0} \cos(\vec{k}\cdot\vec{X})$ is marginal for $\omega^2 + \vec{k}^2 = 1$. This can be written as a sum of two vertex operators, each of which is exactly marginal: $T(X) = \lambda\Big(e^{\omega X^0 + i\vec{k}.\vec{X}} + e^{\omega X^0 - i\vec{k}.\vec{X}}\Big)$. For generic ω, this is not an exactly marginal deformation because of the singular OPE between the two vertex operators. Hence it does not yield a solution to the classical string equations of motion. However, for $\omega = \frac{1}{\sqrt{2}}$, this perturbation is exactly marginal. Without any loss of generality, we can keep only one component of $\vec{k}$ to be non-zero, denoting the corresponding direction $Y \equiv \sqrt{2}\vec{k}\cdot\vec{X}$. Thus we have

$$\begin{aligned} T(X) &= 2\lambda e^{X^0/\sqrt{2}} \cos\left(\frac{Y}{\sqrt{2}}\right) = \lambda\Big(e^{\frac{X^0+iY}{\sqrt{2}}} + e^{\frac{X^0-iY}{\sqrt{2}}}\Big) &(3.1)\\ &= \lambda(e^U + e^V)\,, &(3.2)\end{aligned}$$

where we have defined new variables

$$U = \frac{X^0 + iY}{\sqrt{2}}\,, \qquad V = \frac{X^0 - iY}{\sqrt{2}}\,, \tag{3.3}$$

such that

$$\begin{aligned} \langle U(z)U(w)\rangle &= \langle V(z)V(w)\rangle = \log|z-w| + \log|z\bar{w} - 1|\ )\,, &(3.4)\\ U(z)V(w) &\sim \text{regular}\,. &(3.5)\end{aligned}$$

Thus U and V behave as commuting time-like coordinates, with no mixing between U and V.

Using the variables U and V, the calculation of $\mathcal{B}$ and $\mathcal{A}^{\mu\nu}$ proceeds very similarly to the calculation in the last section. To compute $\mathcal{B}$, we need to compute

$$\mathcal{B}(u,v) = \langle e^{\lambda\int dt(e^{u+U'} + e^{v+V'})}\rangle\,, \tag{3.6}$$

where u and v are linear combinations of the zero modes of X^0 and Y

$$u = \frac{x^0 + iy}{\sqrt{2}}\,, \qquad v = \frac{x^0 - iy}{\sqrt{2}}\,, \tag{3.7}$$

and U' and V' are non-zero modes of U and V. Since there is no mixing between U and V and (3.6) factorizes as

$$\mathcal{B}(u,v) = \langle e^{\lambda e^u \int dt e^{U'}}\rangle\langle e^{\lambda e^v \int dt e^{V'}}\rangle = f(u)\ f(v)\,, \tag{3.8}$$

where, as in the previous section, we have defined $f(u)$ as

$$f(u) = \frac{1}{1 + 2\pi\lambda e^u}\,. \tag{3.9}$$

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The calculations for $\mathcal{A}^{\mu\nu}$, with the insertion of a graviton vertex operator factorize similarly. For example

$$\begin{aligned}
\mathcal{A}^{uu}(x) &= 2\langle : \partial U(0)\bar{\partial}U(0) : e^{\lambda\int dt(e^{u+U'}+e^{v+V'})}\rangle \\
&= 2\left\langle \left({}^{\circ}_{\circ}\partial U(0)\bar{\partial}U(0){}^{\circ}_{\circ} - \frac{1}{2}\right) e^{\lambda e^{U}\int dt e^{U'}} \right\rangle \langle e^{\lambda e^{v}\int dt e^{V'}}\rangle \\
&= \Big(f(u)-2\Big)\ f(v)\,.
\end{aligned} \tag{3.10}$$

The remaining components give,

$$\begin{aligned}
\mathcal{A}^{uv}(u,v) &= 0\,, \\
\mathcal{A}^{vv}(u,v) &= f(u)\ \ (f(v)-2) \\
\mathcal{A}^{ij}(u,v) &= \delta_{ij} f(u)\ f(v)\,.
\end{aligned}$$

Furthermore, from the relations

$$A_{uu} = \frac{1}{2}(A_{00}-A_{yy}+2iA_{0y});\ \ A_{uv} = \frac{1}{2}(A_{00}+A_{yy});\ \ A_{vv} = \frac{1}{2}(A_{00}-A_{yy}-2iA_{0y})\,, \tag{3.11}$$

and using (2.5), the stress tensor for this spatially inhomogenous decaying solution can be calculated to be

$$T_{00}(x^0,y) = -\mathcal{T}_p\ \frac{1+2\pi\lambda e^{x^0/\sqrt{2}}\cos(y/\sqrt{2})}{1+4\pi\lambda e^{x^0/\sqrt{2}}\cos(y/\sqrt{2})+4\pi^2\lambda^2 e^{\sqrt{2}x^0}}\,, \tag{3.12}$$

$$T_{0y}(x^0,y) = \mathcal{T}_p\ \frac{-2\pi\lambda e^{x^0/\sqrt{2}}\sin(y/\sqrt{2})}{1+4\pi\lambda e^{x^0/\sqrt{2}}\cos(y/\sqrt{2})+4\pi^2\lambda^2 e^{\sqrt{2}x^0}}\,, \tag{3.13}$$

$$T_{yy}(x^0,y) = -T_{00}(x^0,y)\,. \tag{3.14}$$

This stress tensor is conserved, i.e.

$$\partial_0 T^{00} - \partial_y T^{y0} = 0\,, \tag{3.15}$$

The form of the stress tensor and its late time behavior is qualitatively different from that obtained in the spatially homogenous case in section 2.2. Certainly at large times $x^0 \to \infty$ *all* components of the stress tensor (3.12)–(3.14), including the energy density, approach zero. However, this result probably cannot be trusted since, at a finite critical time $x^0_c \equiv \sqrt{2}\ln(1/(2\pi|\lambda|))$, the stress energy tensor exhibits singularities at the spatial loci

$$y_n = 2\sqrt{2}n\pi,\ n\in\mathbb{Z}\,, \qquad (\lambda<0)\,, \tag{3.16}$$

$$y_n = 2\sqrt{2}(n+\frac{1}{2})\pi,\ n\in\mathbb{Z}\,, \qquad (\lambda>0)\,. \tag{3.17}$$

These singularities are what one would expect since, for some values of y, we are perturbing with a negative tachyon and, according to (2.26), such perturbations give rise to singularities at finite time already in the spatially homogenous setting. In [4] Sen proposed that these singularities should be interpreted as codimension one D-branes. To see this, we introduce the auxiliary variable

$$\Delta = e^{\frac{x^0 - x_c^0}{\sqrt{2}}}\,, \tag{3.18}$$

and, for either sign of λ, we write the energy density $\rho = -T_{00}$ as

$$\rho = \mathcal{T}_p\, \frac{(1-\Delta) + 2\Delta\sin^2((y-y_0)/2\sqrt{2})}{(1-\Delta)^2 + 4\Delta\sin^2((y-y_0)/2\sqrt{2})} \overset{\Delta\to 1}{\simeq} \mathcal{T}_p\, \left[\frac{1}{2} + \sqrt{2}\pi\sum_{n\in\mathbb{Z}} \mathrm{sgn}(x_c^0 - x^0)\delta(y-y_n)\right]. \tag{3.19}$$

The limit was computed using $\lim_{\epsilon\to 0}\frac{\epsilon}{\epsilon^2+\alpha^2} = \pi\delta(\alpha)$ close to each singular locus. The corresponding average energy density changes discontinuously from $\mathcal{T}_p$ to 0 as we pass x_c^0. The form of the limiting energy density (3.19) suggests that the missing energy forms co-dimension one defects at the y_n. As we pass the critical time x_c^0, the loss in energy at each y_n is $2\sqrt{2}\pi\mathcal{T}_p$. This result for the defect energy can be verified using energy conservation, noting that the defects are $\Delta y = 2\sqrt{2}\pi$ apart, and no bulk energy remains after they form. A co-dimension one D-brane has tension $\mathcal{T}_{p-1} = 2\pi\mathcal{T}_p$ and so the defect has additional energy, beyond that needed to form a D-brane. Nevertheless it is plausible that these defects are indeed related to D-branes since the spatial potential $T(X) \propto \cos(Y/\sqrt{2})$ tends to confine the ends of the strings, much as in the corresponding off-shell discussion in [18]).

4. Rolling tachyons in superstring theory

The purpose of this section is to generalize the results of the previous sections to the superstring. In each step of the computation details are modified, and so must be repeated; but the final results are closely analogous to the bosonic case.

4.1 Generalities

In the superstring case world-sheet fermions must be included in a manner consistent with world-sheet supersymmetry. A convenient way implement this is to introduce world-sheet superfields. Thus the string coordinates (on the boundary of the disk) are represented by

$$\mathbf{X}^\mu = X^\mu + \theta\psi^\mu\,, \tag{4.1}$$

and the Chan-Paton index of the brane is encoded in the boundary fermions

$$\Gamma^I = \eta^I + \theta F^I\,. \tag{4.2}$$

We will consider only the simplest case of a single non-BPS D-brane. Since this corresponds to a single boundary fermion we can omit the index I. In this formalism the boundary action for a general tachyon profile $T(\mathbf{X})$ is

$$I_{\rm bndy} = \int dt d\theta[\mathbf{\Gamma} D\mathbf{\Gamma} + \mathbf{\Gamma} T(\mathbf{X})]\,, \tag{4.3}$$

where D denotes the derivative in superspace $D = \partial_\theta + \theta\partial_z$. A single boundary fermion $\mathbf{\Gamma}$ can be integrated out with the result

$$\langle e^{-I_{\rm bndy}}\cdots\rangle = \left\langle P\exp\left[-\int dtd\theta\Gamma T(\mathbf{X})\right]_{\Gamma-{\rm even}}\cdots\right\rangle = \left\langle P\cosh\left[\int dtd\theta T(\mathbf{X})\right]\cdots\right\rangle\,, \tag{4.4}$$

within correlators. Here P is the standard path-ordering operator. Note that this path-ordering operator is not trivial in the above espression because $\int dtd\theta T(\mathbf{X})$ is fermionic and then it does not commute with itself. The boundary fermions serve to make world-sheet supersymmetry manifest but, in the present context, they play the role of Chan-Paton matrices σ_1, for which the restriction to even terms arises from the overall trace.[6]

Following the bosonic example, our main interest is in inserting the identity operator

$$\mathcal{B}(x) = \left\langle P\cosh\left(\int dtd\theta T(\mathbf{X})\right)\right\rangle, \tag{4.5}$$

and the gravity vertex operator

$$\mathcal{A}^{\mu\nu} = \left\langle V^{\mu\nu}(0,0)P\cosh\left(\int dtd\theta T(\mathbf{X})\right)\right\rangle, \tag{4.6}$$

where, in the present case,

$$V^{\mu\nu}(0,0) = 2\int d\theta d\bar\theta[D\mathbf{X}^\mu\bar D\mathbf{X}^\nu]_{z=\bar z=0} \tag{4.7}$$

The energy momentum tensor still follows from (2.5).

In (4.5) and (4.6) the brackets $\langle\cdots\rangle$ denote averaging with respect to the non-zero mode part of the bosonic fields, as before. In concrete examples, we can evaluate these expressions in perturbation theory using the two point function [20]

$$\langle\mathbf{X}^\mu\mathbf{X}^\nu\rangle = -\eta^{\mu\nu}\log|z_{12}|^2\,, \tag{4.8}$$

where

$$z_{12} = z_1 - z_2 - i\sqrt{z_1 z_2}\theta_1\theta_2\,. \tag{4.9}$$

The form (4.8) of the two point function is valid when both coordinates are on the boundary of the disk. This will suffice for our applications.

4.2 The simple decay

We consider first the supersymmetric version of the profile (3.1), i.e.

$$T(\mathbf{X}) = \lambda e^{\mathbf{X}^0/\sqrt{2}}\,. \tag{4.10}$$

[6]We have ignored the contact term $e^{-\int dtd\theta T(\mathbf{X})^2}$ which appear in (4.4) for general tachyon profiles. This term is important in BSFT discussions of tachyon condensation [14], as well as in the time dependent case [19]. Here we follow Sen [2] and regard the right hand side of (4.4) as the starting point of our discussions.

Expanding (4.5) in the parameter λ and using (4.8) yields

$$\mathcal{B}(x^0) = \sum_{n=0}^{\infty}(-1)^n \left(2\pi\lambda e^{x^0/\sqrt{2}}\right)^{2n} \int \prod_{i=1}^{2n} \frac{dt_i}{2\pi} d\theta_i \; \Theta(t_1 - t_2)\Theta(t_2 - t_3)\cdots\Theta(t_{2n-1} - t_{2n}) \times$$
$$\times \prod_{i<j} |e^{it_i} - e^{it_j} - ie^{\frac{i}{2}(t_i+t_j)}\theta_i\theta_j| \,. \tag{4.11}$$

Noticing $|e^{it_i} - e^{it_j} - ie^{\frac{i}{2}(t_i+t_j)}\theta_i\theta_j| = |e^{it_i} - e^{it_j}| + \mathrm{sign}(t_i - t_j)\theta_i\theta_j$, the integrals can be evaluated with the result[7]

$$\int \prod_{i=1}^{2n} \frac{dt_i}{2\pi} d\theta_i \; \Theta(t_1 - t_2)\Theta(t_2 - t_3)\cdots\Theta(t_{2n-1} - t_{2n}) \prod_{i<j} |e^{it_i} - e^{it_j} - ie^{\frac{i}{2}(t_i+t_j)}\theta_i\theta_j| = \frac{1}{2^n} \,, \tag{4.12}$$

so, after summation of the series, we find

$$\mathcal{B}(x^0) = \frac{1}{1 + 2\pi^2\lambda^2 e^{\sqrt{2}x^0}} \,. \tag{4.13}$$

The insertion of a graviton operator brings a few more complications, as in the bosonic case. The proper normal ordering again gives

$$: V^{\mu\nu} : \;=\; {}^{\circ}_{\circ} V^{\mu\nu} {}^{\circ}_{\circ} + \eta^{\mu\nu} \,, \tag{4.14}$$

and thus, without any further effort,

$$\mathcal{A}^{ij}(x^0) = \mathcal{B}(x^0)\delta^{ij} \,. \tag{4.15}$$

The graviton with two temporal indices is evaluated in perturbation theory starting from (4.6). In addition to a term $\mathcal{B}\eta^{00} = -\mathcal{B}$ from the normal ordering (4.14), we find an integral over the correlator

$$\left\langle \left(\int d\theta d\bar{\theta} {}^{\circ}_{\circ} D\mathbf{X}^{\mu}(w)\bar{D}\mathbf{X}^{\nu}(\bar{w}) {}^{\circ}_{\circ} \right) \prod_{i=1}^{2n} e^{\mathbf{X}(z_i)/\sqrt{2}} \right\rangle =$$
$$= \prod_{i<j} |z_i - z_j - i\sqrt{z_i z_j}\theta_i\theta_j| \frac{1}{2} \sum_{k,l} \frac{1}{z_k - w} \, \frac{1}{\bar{z}_l - \bar{w}} \,, \tag{4.16}$$

with $w = \bar{w} = 0$. The expressions can then be combined as

$$\mathcal{A}^{00} = -\mathcal{B} + 2\sum_{n=0}^{\infty}(-1)^n(2\pi\lambda e^{x^0/\sqrt{2}})^{2n} \; I_{2n} \,, \tag{4.17}$$

where the integrals[8]

$$I_{2n} = \int \prod_{i=1}^{2n} \frac{dt_i}{2\pi} d\theta_i \; \Theta(t_1 - t_2)\cdots\Theta(t_{2n-1} - t_{2n}) \prod_{i<j} |e^{it_i} - e^{it_j} - ie^{\frac{i}{2}(t_i+t_j)}\theta_i\theta_j| \times$$
$$\times \frac{1}{2} \sum_{k,l} e^{i(t_k - t_l)}\theta_k\theta_l = \frac{1}{2^n} \; \delta_{n>0} \,. \tag{4.18}$$

[7] We checked (4.12) for $n \leq 3$.

[8] We also checked (4.18) for $n \leq 3$.

The final result thus becomes

$$\mathcal{A}^{00}(x^0) = \frac{1}{1+2\pi^2\lambda^2 e^{\sqrt{2}x^0}} - 2 = \mathcal{B}(x^0) - 2\,, \tag{4.19}$$

as in the bosonic case.

The non-vanishing components of the energy momentum tensor now read

$$T_{00} = K(-\mathcal{B}(x^0) + \mathcal{A}_{00}(x^0)) = -2K\,, \qquad T_{ij} = K(\mathcal{B}(x^0)\delta_{ij} + \mathcal{A}_{ij}(x^0)) = 2K\delta_{ij}\mathcal{B}(x^0)\,, \tag{4.20}$$

where $\mathcal{B}$ was given in (4.13). The overall constant K again is identified as $K = \frac{1}{2}\mathcal{T}_p$. As in the bosonic case, the energy density is constant $\rho = \mathcal{T}_p$ throughout the decay. The pressure

$$p = 2\mathcal{B}(x^0) = \frac{\mathcal{T}_p}{1+2\pi^2\lambda^2 e^{\sqrt{2}x^0}}\,, \tag{4.21}$$

is equal to the energy density $p = \rho$ for the unstable brane at $x^0 = -\infty$; but it decays exponentially to zero at large times. The main difference with the bosonic case is that now the decay is symmetric under $\lambda \to -\lambda$. In the supersymmetric case there is no singularity for either sign, as expected since the tachyon potential is symmetric, with both directions sloping down to the stable closed string vacuum.

All these results are closely analogous to Sen's discussions, based on the potential $T(X) = \lambda\cosh(X)$.

4.3 Spatial inhomogeneity

We also want to consider a spatially inhomogeneous profile for the superstring. The simplest example is

$$T(\mathbf{X}) = 2\lambda\; e^{\frac{1}{2}\mathbf{X}^0} \cos\left(\frac{1}{2}\mathbf{Y}\right), \tag{4.22}$$

where Y is one of the spatial directions. As in the bosonic case, this example is factorizable

$$T(\mathbf{X}) = \lambda e^{\frac{1}{2}(\mathbf{X^0}+i\mathbf{Y})} + \lambda e^{\frac{1}{2}(\mathbf{X^0}-i\mathbf{Y})}\,, \tag{4.23}$$

where, crucially, $\mathbf{X^0}+i\mathbf{Y}$ and $\mathbf{X^0}-i\mathbf{Y}$ have regular OPEs. Thus the example is essentially two copies of the profile (4.10).[9] From (4.13) we immediately find

$$\mathcal{B}(x^0,y) = \frac{1}{2}\mathcal{T}_p\,\frac{1}{|1+2\pi^2\lambda^2 e^{x^0+iy}|^2}\,, \tag{4.24}$$

while (4.15) gives $\mathcal{A}^{ij} = \delta^{ij}\mathcal{B}$ for $i,j \neq y$, and (4.17) combines with (4.13) to give

$$\mathcal{A}_{\frac{1}{\sqrt{2}}(x^0+iy),\frac{1}{\sqrt{2}}(x^0+iy)} = \frac{1}{2}\mathcal{T}_p\left(\frac{1}{1+2\pi^2\lambda^2 e^{x^0+iy}} - 2\right)\frac{1}{1+2\pi^2\lambda^2 e^{x^0-iy}}\,. \tag{4.25}$$

[9] The factorization could be imperfect in the superstring case, due to the fermionic nature of the boundary fermions [4]. This may spoil the exact marginality of the profile. However, we expect that it indeed factorizes since there is a path-ordered operator P in the definition of the correators and this P may recover the the marginality of the profile.

The expressions, along with the complex conjugate of the last equation, yields the energy momentum tensor

$$T_{00} = -\mathcal{T}_p \,\mathrm{Re}\frac{1}{1+2\pi^2\lambda^2 e^{x^0-iy}} = -\mathcal{T}_p \,\frac{1+2\pi^2\lambda^2 e^{x^0}\cos y}{1+4\pi^2\lambda^2 e^{x^0}\cos y + 4\pi^4\lambda^4 e^{2x^0}}\,, \tag{4.26}$$

$$T_{yy} = -T_{00}\,, \tag{4.27}$$

$$T_{0y} = \mathcal{T}_p \,\mathrm{Im}\frac{1}{1+2\pi^2\lambda^2 e^{x^0-iy}} = \mathcal{T}_p \,\frac{2\pi^2\lambda^2 e^{x^0}\sin y}{1+4\pi^2\lambda^2 e^{x^0}\cos y + 4\pi^4\lambda^4 e^{2x^0}}\,. \tag{4.28}$$

The energy momentum tensor again exhibits singularities. They appear at the critical time $x^0_c = -\log(2\pi^2\lambda^2)$ and at the loci

$$y_n = (2n+1)\pi\,, \qquad n \in \mathbb{Z}\,. \tag{4.29}$$

The energy density $\rho = -T_{00}$ behaves as

$$\rho \to \mathcal{T}_p \left[\frac{1}{2} + 2\pi \sum_{n\in Z} \mathrm{sgn}(x^0_c - x^0)\delta(y - y_n)\right]\,, \tag{4.30}$$

as the critical time is approached, essentially like the bosonic case (3.19). In the superstring case the interpretation is on a firmer footing since the tachyon profile (4.22) amounts to the rolling down either of two sides of a symmetric, and regular, potential. Given the topology of this situation it is not at all surprising that codimension one defects result. Additionally, defects interpolating between the two sides of the potential are known to couple to *RR*-fields such that, in the present case, consecutive branes have opposite signs, *D*-branes and anti-*D*-branes. They are sufficiently separated that the low energy fluctuation spectrum contains no tachyons; so the configuration is classically stable. Nevertheless, the energy density of the defect is larger by a factor of $\sqrt{2}$ than that of a *BPS* D-brane. As in the bosonic case we can verify this result using energy conservation.

5. Symmetries and boundary states

The purpose of this section is to reconsider the tachyon profiles in the previous sections using the methods and results from Sen's recent work on related profiles [1].

5.1 The group of time-dependent marginal deformations

A natural set of spatially homogeneous tachyon profiles in bosonic string theory is

$$T(X) = \lambda_1 \cosh X^0 + \lambda_2 \sinh X^0\,, \tag{5.1}$$

for general (λ_1, λ_2). These tachyon profiles are invariant under $X^0 \to X^0 + c, (\lambda_1 \pm \lambda_2) \to (\lambda_1 \pm \lambda_2)e^{\mp c}$; so time translations act on the parameters (λ_1, λ_2) as the group SO(1,1). This means group invariants $\lambda_1^2 - \lambda_2^2$ and $\mathrm{sign}(\lambda_1 \pm \lambda_2)$ classify the possible perturbations, in the sense that any two tachyon profiles of the form (5.1) with identical values of these invariants are physically equivalent.

As a representative of the *elliptic* equivalence class $\lambda_1^2 - \lambda_2^2 > 0$, we can take $T(X) = \lambda_1 \cosh X^0$. Since $T(X^0 = 0) = \lambda_1$, $\partial_0 T(X^0 = 0) = 0$ this corresponds to there being a time, chosen without loss of generality as $X^0 = 0$, where the tachyon is displaced from the top of potential, but its velocity vanishes. Physically, we can thus think of the elliptic equivalence class as having *negative energy*. The tachyon field starts at the bottom of the potential, reaches a maximum at an intermediate time taken as $X^0 = 0$, and it returns to its starting point at large times. The sign of λ_1 determines which side of the potential the entire trajectory takes place, with positive lambda corresponding to the stable side for bosonic strings.

As representative of the *hyperbolic* equivalence class $\lambda_1^2 - \lambda_2^2 < 0$, we can take $T(X) = \lambda_2 \sinh X^0$, also considered in [1]. For this profile $T(X^0 = 0) = 0$, $\partial_0 T(X^0 = 0) = \lambda_2$; so this corresponds to there being a time where the tachyon is on top of the potential, with a non-vanishing velocity. Physically we can think of the hyperbolic trajectories as having *positive energy*, with the tachyon field starting at the bottom on one side, reaching the maximum the top of the potential at the time chosen as $X^0 = 0$, and then rolling to the bottom of the potential on the other side. The sign of λ_2 determines which side of the potential the motion starts from.

The main focus in this paper is the *parabolic* equivalence class $\lambda_1^2 = \lambda_2^2$. Taking $\lambda_1 = \lambda_2 (= \lambda)$ in (5.1) gives

$$T(X) = \lambda e^{X^0} . \tag{5.2}$$

A good physical characterization of the parabolic case is *vanishing energy*; the tachyon starts at the top of the potential, reaching the bottom of the potential at late times. Having no energy in the initial state, except for the tension of the unstable brane itself, this profile realizes the intuition of a spontaneously decaying brane. Time translations can be absorbed in the magnitude of the parameter λ which is thus inconsequential. Taking $\lambda_1 = -\lambda_2 (= \lambda)$ would be the time-reversed trajectory, and the sign of λ corresponds to the two sides of the potential. The parabolic tachyon profile is called a half S-brane in [13], with the hyperbolic one being an S-brane.

The parabolic profiles can be obtained as limiting cases of the elliptic ones. Indeed, starting from $T(X) = \lambda_1 \cosh(X^0)$ and taking the limit $\lambda_1 \to 0$, $c \to \infty$ with fixed $\lambda = \frac{1}{2}\lambda_1 e^c$, we recover (5.2). The limit corresponds to tuning the displacement of the tachyon at $X^0 = 0$ to zero, while moving the time at which the maximum is reached from from $X^0 = 0$ to the infinite past.

Since the parabolic profiles can be represented as limits of the elliptic ones, the corresponding energy-momentum tensors can be determined from those computed by Sen [1]. For (5.1), with $\lambda_2 = 0$, Sen found the stress tensor $T_{00} = \frac{T_p}{2}(1 + \cos 2\pi\lambda_1)$ and $T_{ij} = -\delta_{ij} T_{00} f(x^0)$, where

$$f(x^0) = \frac{1}{1 + \sin(\lambda_1 \pi) e^{x^0}} + \frac{1}{1 + \sin(\lambda_1 \pi) e^{-x^0}} - 1 . \tag{5.3}$$

Generalizing this result to arbitrary elliptic (λ_1, λ_2), using the symmetry under time translation, we find

$$T_{00} = \frac{\mathcal{T}_p}{2}\left[1+\cos(2\pi\sqrt{\lambda_1^2-\lambda_2^2})\right],$$
$$T_{ij} = -\delta_{ij}T_{00}f(x^0)\,, \tag{5.4}$$

where

$$f(x^0) = \frac{1}{1+(\lambda_1+\lambda_2)\frac{\sin(\pi\sqrt{\lambda_1^2-\lambda_2^2})}{\sqrt{\lambda_1^2-\lambda_2^2}}e^{x^0}} + \frac{1}{1+(\lambda_1-\lambda_2)\frac{\sin(\pi\sqrt{\lambda_1^2-\lambda_2^2})}{\sqrt{\lambda_1^2-\lambda_2^2}}e^{-x^0}} - 1. \tag{5.5}$$

Taking the limit $\lambda_1 \to \lambda_2^+ (\equiv \lambda)$ to the parabolic case this gives $T_{00} = \mathcal{T}_p$ and $T_{ij} = -\delta_{ij}\mathcal{T}_p f(x^0)$ with

$$f(x^0) = \frac{1}{1+2\pi\lambda e^{x^0}}\,, \tag{5.6}$$

in agreement with our explicit computations. The corresponding limit for the superstring similarly lead to the results found in the previous section.

5.2 Spatial variation

We can also consider the profile with spatial variation

$$T(X) = 2\lambda e^{X^0/\sqrt{2}}\cos\left(\frac{Y}{\sqrt{2}}\right), \tag{5.7}$$

as a limit of the profile

$$T(X) = \lambda_1 \cosh\frac{X^0}{\sqrt{2}}\cos\left(\frac{Y}{\sqrt{2}}\right), \tag{5.8}$$

considered previously by Sen. The procedure is to replace $X^0 \to X^0 + c$ and then taking the limit $\lambda_1 \to 0,\ c \to \infty$ with fixed $\lambda \equiv \frac{1}{4}\lambda_1 e^{c/\sqrt{2}}$. The profile (5.8) was found to yield the energy momentum tensor

$$T_{00} = \mathcal{T}_p\ \mathrm{Re}\tilde{f}\,,$$
$$T_{0y} = \mathcal{T}_p\ \mathrm{Im}\tilde{f}\,,$$
$$T_{yy} = -\mathcal{T}_p\ \mathrm{Re}\tilde{f} = -T_{00}\,,$$
$$T_{ij} = -\mathcal{T}_p\ \delta_{ij}|\tilde{f}|^2\,, \tag{5.9}$$

where

$$\tilde{f} = \frac{1}{1+\sin(\tilde{\lambda}\pi/2)\,e^{(x^0+iy)/\sqrt{2}}} + \frac{1}{1+\sin(\tilde{\lambda}\pi/2)\,e^{-(x^0+iy)/\sqrt{2}}} - 1\,, \tag{5.10}$$

so we can simply take the limit and find

$$\tilde{f} = \frac{1}{1+2\pi\lambda\,e^{(x^0+iy)/\sqrt{2}}}\,. \tag{5.11}$$

Then (5.9) agrees with the results of our explicit computations (3.12)–(3.14).

5.3 The full boundary states

Since the parabolic case can be obtained as a suitable limit of other cases we can also use previous works to obtain the full boundary state.

Let us first review the strategy following [1]. Starting with the general profile (5.1), performing the Wick rotation $X^0 = iX$ and redefining $\lambda_2 = -i\lambda_2'$, we find the tachyon profile $T(X) = \lambda_1 \cos X + \lambda_2' \sin X$. This action contains only modes with integer momentum modes; so we can consider the theory compactified on a self-dual radius $R = 1$, instead of the uncompactified theory. At this radius there is an SU(2) current algebra with zero-modes

$$J^{\pm} = \oint \frac{dz}{2\pi i} e^{\pm 2iX_R(z)}, \qquad J^3 = \oint \frac{dz}{2\pi i} i\partial X_R(z). \tag{5.12}$$

The tachyon profile $T(X)$ is precisely a linear combination of these generators and we see that the (λ_1, λ_2') can be represented as SU(2) parameters as

$$\mathcal{R} = \exp\left(i\pi \begin{pmatrix} 0 & \lambda_1 + i\lambda_2' \\ \lambda_1 - i\lambda_2' & 0 \end{pmatrix}\right) = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}, \tag{5.13}$$

where

$$a = \cos\left(\pi\sqrt{\lambda_1^2 + \lambda_2'^2}\right), \qquad b = i\,\frac{\lambda_1 + i\lambda_2'}{\sqrt{\lambda_1^2 + \lambda_2'^2}}\,\sin\left(\pi\sqrt{\lambda_1^2 + \lambda_2'^2}\right). \tag{5.14}$$

The boundary state for the unperturbed D-brane, with X^0 kept explicit, can be written as [21]

$$|B\rangle_{x^0}^{\text{Neumann}} = \sum_{j=0,\frac{1}{2},\cdots} \sum_{m=j}^{j} |j, m, m\rangle\rangle, \tag{5.15}$$

where $|j, m, m\rangle\rangle$ is the Virasoro-Ishibashi state [22] for the discrete Virasoro primary $|j, m, m\rangle \equiv |j, m\rangle\overline{|j, m\rangle}$. It is simply a sum of all Virasoro descendants of the primary $|j, m, m\rangle$.

Since the tachyon profile $T(X)$ is an element of the SU(2) algebra, and $|j, m\rangle$ transforms in the (j, m) representation of SU(2) algebra, the non-trivial part of the boundary state becomes simply [21, 23, 24]

$$|B\rangle_{x^0} = \sum_{j=0,1/2,\cdots} \sum_{m=j}^{j} D^j_{m,-m}(\mathcal{R}) |j, m, m\rangle\rangle, \tag{5.16}$$

where $D^j_{m,-m}(\mathcal{R})$ is the spin j representation matrix of the rotation $\mathcal{R}$ in J_z eigenbasis (see [24] for an explicit form) .

To find the boundary state for the time-dependent tachyon profile (5.1), we then apply the appropriate inverse Wick rotation noting that, after the inverse Wick rotation, b^* is not the complex conjugate of b. In the parabolic limit $\lambda_1 \to \lambda_2^+ \equiv \lambda$ the "rotation" matrix becomes

$$\mathcal{R} = \begin{pmatrix} 1 & 0 \\ 2\pi i\lambda & 1 \end{pmatrix}. \tag{5.17}$$

These considerations indeed give the correct stress tensor, for general (λ_1, λ_2). Writing the boundary state as

$$|B\rangle_{x^0} = f(x^0)|0\rangle + g_{\mu\nu}(x^0)\alpha^{\mu}_{-1}\overline{\alpha^{\nu}_{-1}}|0\rangle + \cdots\,, \tag{5.18}$$

the stress tensor is given by $T_{\mu\nu}(x^0) = f(x^0)\eta_{\mu\nu} + g_{\mu\nu}(x^0)$, as in section 2. The boundary state $|B\rangle_{x^0} \sim \sum_{j=0,1/2,\dots}(D^j_{j,-j}|j,j,j\rangle\rangle + D^j_{-j,j}|j,-j,-j\rangle\rangle)$, with $|j,\pm j,\pm j\rangle\rangle = (i)^{2j} \times e^{\pm 2ijX(0)}|0\rangle + \cdots$ and $D^j_{j,-j} = (-b^*)^{2j}$,$D^j_{-j,j} = b^{2j}$ indeed lead to the $f(x^0)$ given in (5.5). The $g_{\mu\nu}(x^0)$ is similarly reproduced correctly. From the boundary state point of view the simplification offered by the parabolic case is that the representation matrices take the simple form

$$D^j_{-m,m} = \frac{(j+m)!}{(j-m)!(2m)!}(2\pi i\lambda)^{2m}\delta_{m\geq 0}\,, \tag{5.19}$$

rather than the complex formula given in [24].

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